On the addition of degrees of freedom to force-balanced linkages.

V. van der Wijk and Just L. Herder
University of Twente, Faculty of Engineering Technology,
Lab. of Mechanical Automation and Mechatronics, Enschede, The Netherlands.
v.vanderwijk@utwente.nl, j.l.herder@utwente.nl

Abstract The design of shaking-force balanced linkages can be approached by deriving these linkages from balanced linkage architectures. When desired, a possible step is to add degrees-of-freedom (dof), for instance by substituting a link with a n-dof equivalent linkage for which the balanced design of the other links is not affected. This paper shows how the coupler link of a shaking-force balanced 4R four-bar linkage, applied as a 5R five-bar linkage, can be substituted with an equivalent 2-dof pantograph.

1 Introduction

With the increasing speed of manipulators (i.e. mechanisms, robotics), for instance for pick and place tasks, dynamic properties such as shaking-force balance and shaking-moment balance become increasingly important. Acceleration of mass and inertia of moving parts of balanced manipulators do not cause any forces and moments to the base and surrounding, keeping machine vibrations low.

Instead of balancing a manipulator linkage afterwards, it is advantageous to base the design of the linkage on balance properties to minimize complexity, additional mass, and additional inertia (Van der Wijk et al. (2009)). One approach for this is to compose manipulators from balanced linkage sections such as balanced legs (Arakelian and Smith (1999), Wu and Gosselin (2002)). Another approach is to derive manipulators from inherently balanced architectures (Van der Wijk and Herder (2012a)), i.e. linkage architectures that are balanced due to specific kinematic relations. As long as these kinematic relations are maintained, any change to the linkage can be made without affecting the balance properties, for instance by fixing links together and by replacing links with gears.

This paper shows the possibility of adding degrees-of-freedom (dof) to a force-balanced linkage architecture. The substitution of a link with a 2-dof linkage is investigated for which the balanced design of other links is

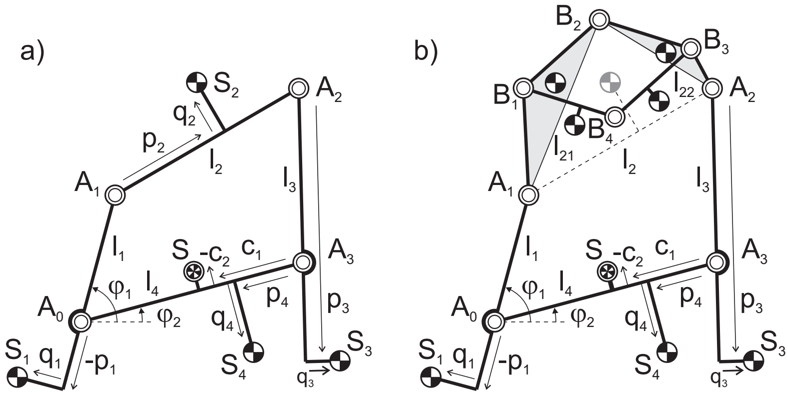

Figure 1. a) Balanced 4R four-bar linkage with CoM at S at link 4; b) Balanced four-bar linkage with coupler l_2 replaced with a pantograph.

not affected. As subject of investigation the coupler link of a shaking-force balanced 4R four-bar linkage, applied as a balanced 5R five-bar linkage, is chosen. First an equivalent model of the coupler link is derived with linear momentum equations and subsequently the conditions for an equivalent 2-dof substitute linkage, a pantograph, are obtained.

2 Equivalent model of coupler link

Figure 1a shows a four-bar linkage $A_0A_1A_2A_3$ of which each link l_i has a mass m_i at its link center-of-mass (link CoM) S_i which are defined with parameters p_i and q_i as indicated. From Berkof and Lowen (1969) and by including the mass m_4 of link 4, the balance conditions for which the CoM of the complete linkage is at an invariant point S in link 4 are written as

$$\begin{aligned} p_1 &= -\frac{m_2(l_2-p_2)}{l_2}\frac{l_1}{m_1} & q_1 &= \frac{m_2q_2}{l_2}\frac{l_1}{m_1} & c_1 &= \frac{1}{m_{tot}}\left(m_1l_4+\frac{m_2(l_2-p_2)}{l_2}l_4+m_4p_4\right) \\ p_3 &= l_3+\frac{m_2p_2}{l_2}\frac{l_3}{m_3} & q_3 &= \frac{m_2q_2}{l_2}\frac{l_3}{m_3} & c_2 &= \frac{1}{m_{tot}}\left(m_4q_4-\frac{m_2q_2}{l_2}l_4\right) \end{aligned} \tag{1}$$

with $m_{tot} = m_1 + m_2 + m_3 + m_4$. The linkage of Fig. 1a then is a force-balanced four-bar linkage when link 4 is stationary with the base and it is a force-balanced five-bar linkage when solely S is stationary with the base as being a movable joint.

To substitute a link without affecting the other links, the substitute linkage has to be equivalent. Therefore first an equivalent model of the coupler link is derived from which the substitute linkage can be found.

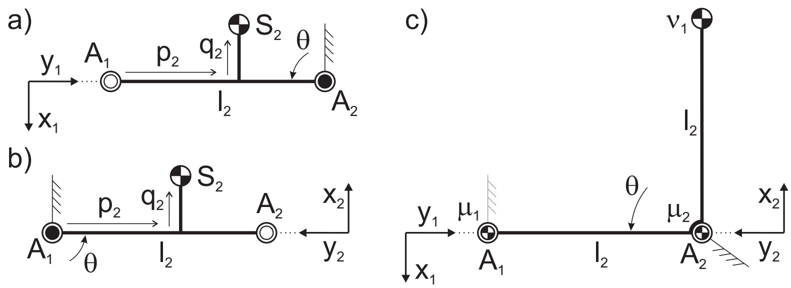

Figure 2. a-b) Coupler link; c) Equivalent Linear Momentum System

With linear momentum equations an equivalent linear momentum system (ELMS) of the coupler can be created. Figures 2a-b show link l_2, of which the linear momentum can be written about A_2 w.r.t. frame x_1y_1 (Fig. 2a) and about A_1 w.r.t. frame x_2y_2 (Fig. 2b) respectively as

$$\frac{\overline{L}_1}{\dot{\theta}} = \begin{bmatrix} m_2(l_2 - p_2) \\ -m_2q_2 \end{bmatrix} \quad \frac{\overline{L}_2}{\dot{\theta}} = \begin{bmatrix} m_2p_2 \\ m_2q_2 \end{bmatrix} \tag{2}$$

Figure 2c shows a model of the coupler with masses μ_1 in A_1, μ_2 in A_2, and mass ν_1 at distance l_2 normal to line A_1A_2 as indicated. Similarly, the linear momentum equations w.r.t. each of the two frames about A_1 and A_2, respectively, can be written as

$$\frac{\overline{L}_1}{\dot{\theta}} = \begin{bmatrix} \mu_1 l_2 \\ -\nu_1 l_2 \end{bmatrix} \quad \frac{\overline{L}_2}{\dot{\theta}} = \begin{bmatrix} \mu_2 l_2 \\ \nu_1 l_2 \end{bmatrix} \tag{3}$$

This implies that the model of Fig. 2c is equivalent to Fig. 2a-b for

$$\mu_1 = \frac{m_2(l_2-p_2)}{l_2} \quad \mu_2 = \frac{m_2p_2}{l_2} \quad \nu_1 = \frac{m_2q_2}{l_2} \tag{4}$$

Also other equivalent models are possible, e.g. with a mass ν_1 at l_2 above A_1. When S_2 is on the line A_1A_2, q_2 is zero for which ν_1 becomes zero too.

3 Equivalent pantograph linkage as substitute

To add one dof, l_2 can be substituted with the 2-dof linkage. Figure 3a shows an ELMS of 2-dof linkage $A_1B_2A_2$ defined with μ_1 in A_1, μ_2 in A_2, and ν_1 at two locations normal to and at equal distance from B_2 as, respectively, l_{21} and l_{22} as indicated. Also here multiple models can be found for equal ELMSs.

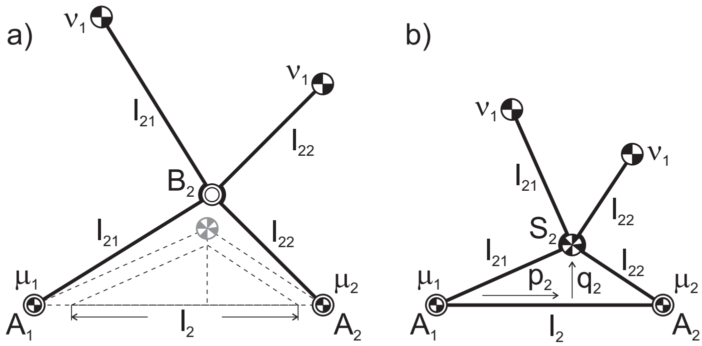

Figure 3. a) Equivalent model of a 2-DoF linkage to replace the coupler; b) Model of the coupler with the CoM of the four masses in S_2.

From Eqs. 1 follows that μ_1, μ_2, and ν_1 need to remain constant to not affect the balance parameters of the other links. Since l_2 is not constant any longer, μ_1, μ_2, and ν_1 can be written as

$$\mu_1 = m_2(1 - \kappa_1) \quad \mu_2 = m_2\kappa_1 \quad \nu_1 = m_2\kappa_2 \tag{5}$$

with $\kappa_1 = p_2/l_2$ and $\kappa_2 = q_2/l_2$ to be constant for any value of l_2. This implies that for all lengths l_2, triangle $A_1A_2S_2$ has to be similar. In general a real linkage $A_1B_2A_2$ cannot generate this similarity. A mechanism which is characterized for its properties of similarity is the pantograph linkage (Artobolevskii (1964)). Figure 1b shows how this linkage can be applied to substitute the coupler link.

Figure 4 shows the substitute pantograph linkage in detail, consisting of four links arranged as parallelogram linkage $B_1B_2B_3B_4$ with each a mass m_{2i} located at distances e_{2i} and f_{2i} from joints B_i as indicated. The total mass of the linkage then is written as $m_2 = m_{21} + m_{22} + m_{23} + m_{24}$. The parallelogram linkage, and in specific joints B_1 and B_3, are defined with principal lengths a_1 and a_2 from B_2, respectively, and with angles α_1 and α_2 with the lines A_1B_2 and B_2A_2, respectively.

The pantograph linkage is equivalent to the coupler link when its CoM is located at S_2 of the similar triangle $A_1A_2S_2$ at all times. To find the conditions for which this holds, the linear momentum of the pantograph linkage can be written to be equal to the linear momentum of the ELMS of Fig. 3a. These equations can be written for each dof individually as shown in Van der Wijk and Herder (2012a). The linear momentum for $\dot{\theta}_1$ with

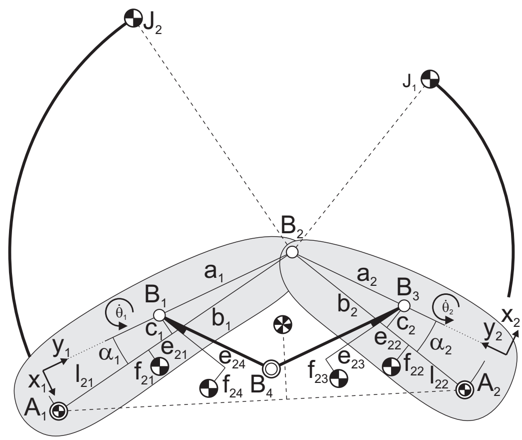

Figure 4. Pantograph linkage $A_1B_1B_2B_3B_4A_2$ and its parameters.

respect to frame x_1y_1 with $\dot{\theta}_2 = 0$ and A_2B_2 fixed can be written as

$$\frac{\overline{L}_{21}}{\dot{\theta}_1} = \begin{bmatrix} u_1 l_{21}\cos\alpha_1 + v_1 l_{21}\sin\alpha_1 \\ u_1 l_{21}\sin\alpha_1 - v_1 l_{21}\cos\alpha_1 \end{bmatrix} = \begin{bmatrix} (m_{21}e_{21}+m_{23}e_{23})\cos\alpha_1 - (m_{21}f_{21}+m_{23}f_{23})\sin\alpha_1 + m_{24}a_1 \\ (m_{21}e_{21}+m_{23}e_{23})\sin\alpha_1 + (m_{21}f_{21}+m_{23}f_{23})\cos\alpha_1 \end{bmatrix} \tag{6}$$

and the linear momentum for $\dot{\theta}_2$ with respect to frame x_2y_2 with $\dot{\theta}_1 = 0$ and A_1B_2 fixed writes

$$\frac{\overline{L}_{22}}{\dot{\theta}_2} = \begin{bmatrix} u_2 l_{22}\cos\alpha_2 + v_1 l_{22}\sin\alpha_2 \\ -u_2 l_{22}\sin\alpha_2 + v_1 l_{22}\cos\alpha_2 \end{bmatrix} = \begin{bmatrix} (m_{22}e_{22}+m_{24}e_{24})\cos\alpha_2 - (m_{22}f_{22}+m_{24}f_{24})\sin\alpha_2 + m_{23}a_2 \\ -(m_{22}e_{22}+m_{24}e_{24})\sin\alpha_2 - (m_{22}f_{22}+m_{24}f_{24})\cos\alpha_2 \end{bmatrix} \tag{7}$$

These equations lead to the resulting four conditions for equivalence

$$\begin{aligned} X_{11}\cos\alpha_1 + X_{12}\sin\alpha_1 &= m_{24}a_1 & X_{21}\cos\alpha_2 + X_{22}\sin\alpha_2 &= m_{23}a_2 \\ X_{11}\sin\alpha_1 - X_{12}\cos\alpha_1 &= 0 & X_{21}\sin\alpha_2 - X_{22}\cos\alpha_2 &= 0 \end{aligned} \tag{8}$$

with

$$\begin{aligned} X_{11} &= u_1 l_{21} - m_{21}e_{21} - m_{23}e_{23} & X_{12} &= v_1 l_{21} + m_{21}f_{21} + m_{23}f_{23} \\ X_{21} &= u_2 l_{22} - m_{22}e_{22} - m_{24}e_{24} & X_{22} &= v_2 l_{22} + m_{22}f_{22} + m_{24}f_{24} \end{aligned} \tag{9}$$

When the mass of each link and the link-CoMs are known, the locations of joints B_1 and B_3 are found with

$$\begin{aligned} \tan\alpha_1 &= \frac{X_{12}}{X_{11}} \qquad a_1 = \frac{1}{m_{24}}(X_{11}\cos\alpha_1 + X_{12}\sin\alpha_1) \\ \tan\alpha_2 &= \frac{X_{22}}{X_{21}} \qquad a_2 = \frac{1}{m_{23}}(X_{21}\cos\alpha_2 + X_{22}\sin\alpha_2) \end{aligned}$$

or by substituting

$$\begin{array}{lll} \cos\alpha_1 = \frac{b_1}{a_1} & \sin\alpha_1 = \frac{c_1}{a_1} & a_1^2 = b_1^2 + c_1^2 \\ \cos\alpha_2 = \frac{b_2}{a_2} & \sin\alpha_2 = \frac{c_2}{a_2} & a_2^2 = b_2^2 + c_2^2 \end{array}$$

in the four conditions of Eqs. 8, which results in

$$\begin{aligned} X_{11}b_1 + X_{12}c_1 &= m_{24}(b_1^2 + c_1^2) \quad & X_{21}b_2 + X_{22}c_2 &= m_{23}(b_2^2 + c_2^2) \\ X_{11}c_1 - X_{12}b_1 &= 0 & X_{21}c_2 - X_{22}b_2 &= 0 \end{aligned} \tag{10}$$

algebraic solutions are obtained for a_1, b_1, and c_1 being

$$a_1 = \frac{1}{m_{24}}\sqrt{X_{11}^2 + X_{12}^2}, \quad b_1 = \frac{X_{11}}{m_{24}}, \quad c_1 = \frac{X_{12}}{m_{24}} \tag{11}$$

and for a_2, b_2, and c_2 being

$$a_2 = \frac{1}{m_{23}}\sqrt{X_{21}^2 + X_{22}^2}, \quad b_2 = \frac{X_{21}}{m_{23}}, \quad c_2 = \frac{X_{22}}{m_{23}} \tag{12}$$

A pantograph linkage according the conditions of Eqs. 8 can replace link l_2 of Fig. 1a as shown in Fig. 1b without affecting the other links for perfect force balance of the complete linkage for all motion.

4 Discussion

In addition to substituting the coupler link, also any of the other three links can be substituted with equivalent linkages to add dofs. Figure 5a shows the result when each of the four links is substituted with an equivalent pantograph with which the mechanism gains four dofs. The procedure to derive the conditions for equivalence is similar to the procedure for the coupler link with some differences due to their specific position within the linkage. Unfortunately this article leaves too few space to discuss them here in detail.

Both branches of the resulting equivalent pantographs can be used, the choice does not affect the design parameters. From Fig. 5a it is observed

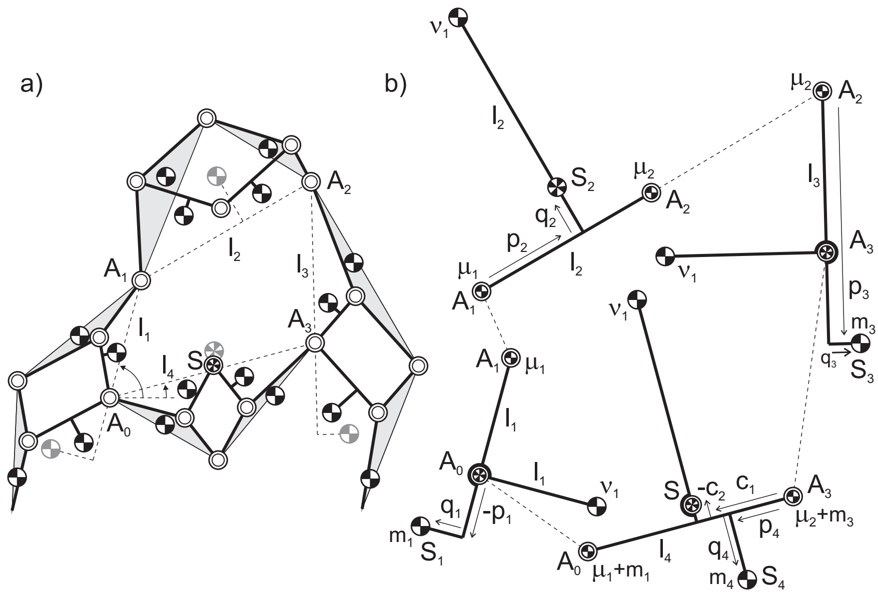

Figure 5. a) Resulting linkage when all four links are substituted with a pantograph gaining four dofs; b) With μ_i and ν_1 the mass of the coupler can be distributed equivalently to the other links. For balancing the linkage then each of the three other links can be separately investigated. For links 1 and 3 the CoM of its link mass and the equivalent masses is in joints A_0 and A_3, respectively. The linkage CoM S at link 4 is the CoM of its equivalent system.

that the coupler substitute pantograph is in the other branch as compared to the other three pantographs.

With the approach in this article it is also possible to substitute links with equivalent 3-dof or higher-dof linkages. These linkages will be pantographic linkages as investigated in Van der Wijk and Herder (2012a), consisting of multiple parallelograms.

The equivalent masses μ_i and ν_1 share another feature which is illustrated in Fig. 5b and which can be obtained from Eqs. 1. With μ_i and ν_1 modeled at link 2 as indicated, S_2 is characterized as the CoM of the three equivalent masses. With μ_1 and ν_1 modeled at link 1 as indicated, A_0 is the CoM of the two equivalent masses and mass m_1. With μ_2 and ν_1 modeled at link 3 as indicated, A_3 is the CoM of the two equivalent masses and mass m_3. With the three equivalent masses modeled at link 4 as indicated and with m_1 in A_0 and m_3 in A_3, S is the CoM of the complete model. μ_i and

ν_1 therefore can be interpreted as the distribution of the mass of link 2 onto the other three links. When linkages become complex, this feature is useful for finding the balance conditions of linkages with arbitrary mass distributions without the need of loop equations, which was shown in Van der Wijk and Herder (2012b). There can be various equivalent models. For instance the initial coupler link can also be modeled as in Fig. 3b for which S_2 is the CoM of the four equivalent masses.

5 Conclusion

For the purpose of adding degrees of freedom (dof) to shaking-force balanced linkages, the coupler link of a 4R four-bar linkage was substituted with a 2-dof equivalent pantograph linkage. An equivalent model of the coupler link was derived with linear momentum equations with which the conditions for the pantograph linkage were determined. It was discussed that the other links can be substituted in a similar way and that also higher-dof equivalent linkages can be used as substitutes. In addition, it was shown how equivalent masses can be used to distribute the coupler mass to the other three links.

Bibliography

V. G. Arakelian and M. R. Smith. Complete shaking force and shaking moment balancing of linkages. *Mechanism and Machine Theory*, 34: 1141–1153, 1999.

I. I. Artobolevskii. *Mechanisms for the Generation of Plane Curves*. Pergamon Press, 1964.

R. S. Berkof and G. G. Lowen. A new method for completely force balancing simple linkages. *J. of Engineering for Industry*, pages 21–26, February 1969.

V. Van der Wijk and J. L. Herder. Synthesis method for linkages with center of mass at invariant link point - pantograph based mechanisms. *Mechanism and Machine Theory*, 48:15–28, 2012a.

V. Van der Wijk and J. L. Herder. Inherently balanced 4r four-bar based linkages. *Proc. of the 13th International Symposium on Advances in Robot Kinematics (ARK), Innsbruck, Austria (accepted)*, 2012b.

V. Van der Wijk, J. L. Herder, and B. Demeulenaere. Comparison of various dynamic balancing principles regarding additional mass and additional inertia. *Journal of Mechanisms and Robotics*, 1(4):04 1006, 2009.

Y. Wu and C. M. Gosselin. On the synthesis of reactionless spatial 3-dof mechanisms using planar four-bar linkages. *Proc. of ASME DETC, Montreal, CA.*, 2002. DETC2002/MECH-34311.